Quadratic Standard Form: $y = ax^2 + bx + c$

Quadratic Factored Form: $y = a(x - r)(x - s)$

We can convert from factored to standard form by expanding and simplifying (FOIL), and from standard to factored by factoring.

Level 1 Factoring

In level 1, the coefficient of x^2 is one: $a = 1$.

1. $y = x^2 - x - 12$

Look for two numbers whose sum is b and whose product is c. Here $b = -1$ and $c = -12$, and the two required numbers are -4 and $+3$. The two numbers can be written 'as is' in the two binomials of the factored form.

$$y = x^2 - x - 12 = (x - 4)(x + 3)$$

2. $y = x^2 - 9x + 14$

Find two numbers that add to -9 and multiply to $+14$; they are -7 and -2.

$$y = x^2 - 9x + 14 = (x - 7)(x - 2)$$

Level 2 Factoring

In level 2, the coefficient of x^2 is different from one, but, if possible, we can make it one by common factoring all terms.

1. $y = 3x^2 + 15x - 72$

b and c are both divisible by a, so we can divide out the common factor 3, then apply the Level 1 method.

$$y = 3x^2 + 15x - 72 = 3(x^2 + 5x - 24) = 3(x + 8)(x - 3)$$

Don't forget to write the common factor in front of everything at each step.

2. $y = -2x^2 - 16x - 30$

If a is negative, make the common factor negative too, because we want to start the Level 1 method with *positive* x^2. Remeber that a negative common factor will change the sign of all terms when divided out.

$$y = -2x^2 - 16x - 30 = -2(x^2 + 8x + 15) = -2(x + 5)(x + 3)$$

Level 3 Factoring

In level 3, the coefficient of x^2 is different from one, but it is not possible to evenly divide b or c by a. We still find two numbers whose sum is b, but now the product must be $a \times c$.

1. $y = 3x^2 + 10x + 8$

Here the sum is $b = +10$ and the product is $a \times c = +24$. The two numbers are $+6$ and $+4$. Next we *decompose* the x term into these two parts: $10x = 6x + 4x$.

$$y = 3x^2 + 10x + 8 = 3x^2 + 6x + 4x + 8$$

Now apply the *factor by grouping* method, where the greatest common factor is found for the first two terms, then again for the second two terms.

$$3x^2 + 6x + 4x + 8 = 3x(x + 2) + 4(x + 2)$$

There are three things to notice at this step. First, the common factor of the second pair of terms must always have a sign, even if it's positive. Second, there are two binomials and they must be identical. If they are different, then there's an algebra error or the two decomposition numbers are wrong. Third, that binomial is a common factor in the two terms of the last step.

Finally, factor the binomial from each term leaving $(3x + 4)$

$$3x(x + 2) + 4(x + 2) = (x + 2)(3x + 4)$$

$$y = 3x^2 + 10x + 8 = (x + 2)(3x + 4)$$

2. $y = 4x^2 + 22x + 10$

No matter what, always look for a common factor.

$$y = 4x^2 + 10x + 16 = 2(2x^2 - 11x + 5)$$

Now apply the Level 3 method inside the brackets: find two numbers whose sum is -11 and whose product is $+10$. They are -10 and -1.

$$2(2x^2 - 11x + 5) = 2(2x^2 - x - 10x + 5)$$

$$= 2\left[x(2x - 1) - 5(2x - 1)\right]$$

$$= 2(2x - 1)(x - 5)$$