1. Two whole numbers differ by 3 and the sum of their squares is 89. What are the numbers?

   Let x be the first number and let y be the second number. Since the two numbers differ by 3, the corresponding equation is

   $$x - y = 3 \, .$$

   The sum of squares statement translates to

   $$x^2 + y^2 = 89 \, .$$

   Many word problems will be very similar to this: you will have a *constraint equation* that relates the variables to one another, and a *quadratic equation*, which will ultimately have to be solved to find your solutions. The problem is that the quadratic often has two different variables—x and y in this case. The next step is to solve the constraint equation for one variable, then replace that variable in the quadratic equation:

   $$y = x - 3 \, .$$

   Substiitute this expression for y into the quadratic equation:

   $$x^2 + (x - 3)^2 = 89 \, .$$

   Now expand, simplify and put into standard form with zero on the right hand side:

   $$x^2 - 3x - 40 = 0 \, .$$

   Factor the quadratic to find the solutions:

   $$(x - 8)\,(x + 5) = 0 \, .$$

   Therefore, $x = 8$ or $x = -5$. Here is where things get a bit tricky. There are two solutions for x, which is just the first number. So there must also be two solutions for the second number y, which are found from the constraint equation $y = x - 3$.

   Solution 1: $x = 8$ and $y = 5$.

   Solution 2: $x = -5$ and $y = -8$.

   Each solution can be checked by substituting x and y into the original quadratic equation.

2. The difference between the length of the hypotenuse and the length of the next longest side of a right triangle is 3 cm. The difference between the two perpendicular sides is 3 cm. Find the three side lengths.

   Let x be the length of the hypotenuse. Then the next longest side is $x-3$ and the shortest side is $(x-3)-3 = x-6$. (Sometimes it fairly straight forward to write everything in terms of one variable without a constraint equation.)

   The quadratic equation will come from the Pythagorean Theorem, $a^2 + b^2 = c^2$. Using the above expressions, it is

   $$(x-3)^2 + (x-6)^2 = x^2 \ .$$

   In standard form, this is

   $$x^2 - 18x + 45 = 0 \ .$$

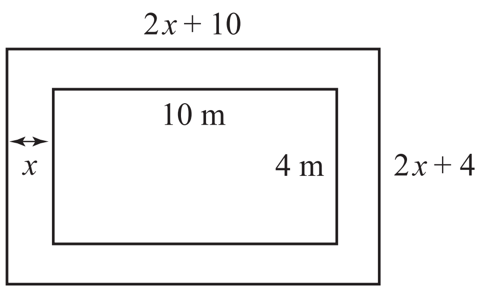
   And in factored form:

   $$(x-15)(x-3) = 0 \ .$$

   Therefore $x = 15$ or $x = 3$. Again we have two solutions for x and will have two solutions for all three sides, x, $x-3$ and $x-6$. For $x = 3$, however, one side length will be zero and the other will be negative, so this solution is inadmissible.

   Solution: Side lengths are 15, 12 and 9.

3. A rectangular swimming pool measuring 10 m by 4 m is surrounded by a deck of uniform width. The combined area of the pool and deck is 72 m^2. What is the width of the deck?



   Let x be the width of the deck. The length of the pool is 10 m, and the width of the deck is x m. Therefore the total length of the deck is $x + x + 10 = 2x + 10$ m. Similarly, the width of the deck is $2x+4$ m. Since the total area of the pool-plus-deck is 72 m^2, the quadratic equation that must be solved is

   $$(2x+10)(2x+4) = 72 \ .$$

After removing a common factor, the standard and factored forms are

$$x^2 + 7x - 8 = 0 \ ,$$

$$(x + 8)\,(x - 1) = 0 \ .$$

Therefore $x = -8$ or $x = 1$. As in example 2, we discard the solution that involves a negative length.

Solution: The width of the deck is 1 m.

Note that in this problem there is a nice way to get to the standard form. Rather than expand the original quadratic, it is possible to remove a common factor from each binomial:

$$2\,(x + 5)\ 2\,(x + 2) = 72$$

$$4\,(x + 5)\,(x + 2) = 72$$

$$(x + 5)\,(x + 2) = \frac{72}{4}$$

$$x^2 + 7x + 10 = 18$$

$$x^2 + 7x - 8 = 0$$