

EXPONENTIAL EQUATIONS FOR WORD PROBLEMS

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	Growth	Decay
1. Doubling time/half-life	$P(t) = P_0 (2)^{\frac{t}{D}}$	$M(t) = M_0 \left(\frac{1}{2}\right)^{\frac{t}{H}}$
2. General growth/decay	$P(t) = P_0 (1 + r)^t$	$V(t) = V_0 (1 - r)^t$
3. Compound interest	$A(t) = A_0 \left(1 + \frac{r}{n}\right)^{nt}$	

TYPES OF QUESTIONS:

1.
  - The number of items in a population  $P(t)$  doubles every  $D$  time units; populations can be people, rabbits, bacteria etc. Time can be measured in years, days, minutes etc.  $P_0$  is the initial population.
  - A mass  $M(t)$  of radioactive material decays to half of its initial value every  $H$  time units; materials are named by isotope like carbon-14, polonium-210, iodine-131 and mass is measured in kilograms, grams, milligrams etc.  $M_0$  is the initial mass.
  - **Note:** the base of each equation reflects the doubling or halving of a quantity in each scenario. If a population is tripled in a time interval, then the base would be 3 and  $D$  would be the tripling time. Base  $\frac{1}{2}$  is a special case and other decay rates work better with the equation in line 2.
  - **Note:** The value of  $t/D$  represents the number of doubling periods spanned in time  $t$ , and the value of  $t/H$  represents the number of half-lives spanned in time  $t$ .
2.
  - A population  $P(t)$  grows at a rate of  $R\%$  in one time unit.  $r$  in the equation must be expressed as a decimal,  $r = R/100$ .  $P_0$  is the initial value.
  - The dollar value  $P(t)$  of an asset *appreciates* by  $R\%$  every year (in questions involving dollar values, time is usually measured in years).  $P_0$  is the initial value.
  - The dollar value  $V(t)$  of an asset *depreciates* by  $R\%$  every year.  $r$  in the equation must be expressed as a decimal,  $r = R/100$ .  $V_0$  is the initial value of the asset.

- The *total percentage* of something  $V(t)$  is reduced by  $R\%$  in one unit of something. Examples: light intensity under water is reduced by 8% for every metre of depth; the colour in a garment fades by 1.5% in every wash. The initial value in a percentage question is always 100.
- 3.
- $A(t)$  is the dollar value of an investment that earns compound interest,  $t$  years after the initial deposit.  $r$  is the decimal equivalent of the annual interest rate.  $n$  is the number of compounding periods in one year.  $A_0$  is the initial value of the investment.
  - Example: An investment of \$1000 earns 3.5% interest compounded quarterly.  $A_0 = 1000$ ,  $r = 0.035$ ,  $n = 4$ .
  - **Note:** the value of  $nt$  represents the total number of compounding periods in the time  $t$ .