Many quadratics word problems require you to find a quadratic equation to analyze or solve and to find a *constraint* equation in the process.

The constraint is needed because there can be only one variable in the quadratic equation, and some questions will start out with two, like area of a rectangle or triangle (length and width, base and height) or something involving two numbers.

The constraint is a relation between the two variables, for example, a perimeter equation or the sum or difference of two numbers. The idea is to solve for one of the variables then substitue that expression into the quadratic equation, resulting in a similar equation with only one variable.

Finding a line or curve of best fit involves minimizing the sum of the squared distances between data and model. Minimizing a sum of squares is an important problem in many areas of mathematics, statistics, physical science and finance.

Question: The sum of two numbers is 28. Find the numbers if the sum of their squares is a minimum.

The first sentence represents the constraint—a relation between the two numbers. The phrase "sum of their squares" is the quadratic function (because they are squared).

Solution: Let x be one number and let y be the other number. The constraint is

$$x + y = 28$$

Solve for one variable:

$$y = 28 - x$$

The function is

$$f = x^2 + y^2$$

Substitute the constraint for the variable y:

$$f(x) = x^2 + (28 - x)^2$$

Expand and simplify:

$$f(x) = 2x^2 - 56x + 784$$

Since we want to find the minimum, we will complete the square to get the equation in vertex form:

$$f(x) = 2(x - 14)^2 + 392$$

Therefore $x = 14$ and $y = 28 - x = 14$. The two numbers are 14 and 14 and the sum of squares is 392.

Note that, because we found the minimum, any other pair of numbers will have a greater sum of squares.

Example 1: $x = 20$, $y = 28 - 20 = 8$, $x^2 + y^2 = 464$.

Example 2: $x = 30$, $y = 28 - 30 = -2$, $x^2 + y^2 = 904$.