One important application of composite functions is transformations. The general equation for transforming a function $f(x)$ into a new function $h(x)$ is:

$$h(x) = af\left[k\left(x - d\right)\right] + c$$

where

- a controls vertical stretches and compressions
- c controls vertical translations
- k controls horizontal stretches and compressions
- d controls horizontal translations

Written in terms of a composition with $g(x) = k(x - d)$, the general equation becomes:

$$h(x) = af\left[g(x)\right] + c$$

1. Use Desmos to explore transformations of the following base functions:

   (a) $f(x) = x^2$
   (b) $f(x) = \sqrt{x}$
   (c) $f(x) = \dfrac{1}{x}$

   Transform the functions by trying different values for each of the parameters a, b, c and d in the general equation shown above.

   Examples:

   $$g(x) = -2\left(x + 3\right)^2 \ + \ 5$$

   $$g(x) = 3\sqrt{-\frac{1}{4}(x - 2)} \ + \ 1$$

   $$g(x) = \frac{-1}{2x - 5} \ - \ 2$$

2. Let $f(x) = x^2 - 3x + 5$ and $g(x) = \dfrac{1}{(x - 2)}$. Evaluate:

   (a) $f(g(4))$

(b) $g(f(1))$

(c) $f(f(-1))$

3. Let $f(x) = x - 2$ and $g(x) = (x-1)^2$. Determine a simplified algebraic model for each composite function.

   (a) $y = f(g(x))$

   (b) $y = g(f(x))$

   (c) $y = g(g(x))$

4. State the domain and range of each composite function in question 3.

5. Let $f(x) = x^2 + 2$ and $g(x) = \sqrt{\frac{3}{2}x}$.

   (a) Find the domain and range of $f \circ g(x)$ and $g \circ f(x)$.

   (b) Check your answers from (a) with Desmos.

   (c) For what value(s) of x is $f(x)$ equal to $f \circ g(x)$?