1. It costs C dollars per kilometre to operate a GO bus, where

$$C = \frac{80}{v} + \frac{v}{80}$$

and v is the average speed in km/h. Determine the average speed at which the bus should be operated to minimize the operating cost. What is the minimum operating cost?

2. Three hundred metres of fencing is available to enclose a rectangular field. The field will also be divided down the centre, using the available fencing, into two smaller rectagular plots. What should the dimensions of the field be for the area to be a maximum?

3. Find the lengths of the sides of an isosceles triangle which has a perimeter of 12 metres and a maximum area.

4. A small window in a cottage is in the form of a rectangle with an equilateral triangle on top. Each side-length of the triangle is equal to the width of the rectangle. If the perimeter is 600 cm, find the maximum area of the window.

5. A rectangular open-topped box is to be made from a piece of cardboard 18 cm by 48 cm by cutting a square from each corner and folding up the sides. What size squares must be removed to maximize the capacity of the box?

6. A right circular cone is inscribed in a sphere of radius 15 cm. Find the dimensions of the cone that has the maximum volume.

7. Find the dimensions and area of the largest rectangle which has its two lower vertices on the x-axis and its two upper vertices on the parabola $y = -x^2 + 16$.

8. Find the point on the graph of $y = \frac{x^2}{\sqrt{2}}$ that is closest to the point (30,0). What is the minimum distance?

9. Electric power for a cottage on one bank of a straight river 200 m wide must come from a power station on the opposite bank of the river and 500 m downstream. If it costs twice as much to lay cable underwater as it does on land, what path should be chosen to minimize the cost?

10. Two isolated cottages are located 12 km apart on a straight country road that runs parallel to the main highway 20 km away. The power company decides to run a wire from the highway to a junction box and from there, wires of equal length to each cottage. Where should the junction be placed to minimze the length of wire needed?

11. A remote control boat is launched from the shore of a pond and travels north at 5 cm/s. At the same time, a duck starts to swim from a point $8\sqrt{2}$ m northeast of the boat and heads west at 7 cm/s. What is the smallest distance between the boat and the duck?

12. At 11:00 a large jet airliner is travelling east at 800 km/h. At the same instant a smaller plane is 45 km east and 90 km north of the airliner. It is at the same altitude and heading south at 600 km/h. What is the closest that the two planes will be and at what time does that happen?

13. A real estate firm owns 160 studio apartments that can be rented for $1000 per month. For each $50 increase in rent, there are four vacancies created that cannot be filled. What should the monthly rent be to maximize the firm's revenue? What is the maximum revenue.

14. A tire company sells tires for cars at $120 each. They give a $5 discount for every one hundred tires purchased. Find the number of tires the company should persuade Canadian Tire to purchase in order to make the most income?

15. A power station is on the bank of a straight river that is 150 m wide. A factory is on the other side of the river 800 m downstream from the power station. A cable, which costs $100/m to place under the water and $60/m on land, is to be run from the power station to the factory. What path should be chosen to minimize the cost?

16. Find the point on the parabola $y = x^2 + \frac{9}{4}$ that is closest to (3,0).

17. A juice can in the shape of a right circular cylinder has a fixed capacity. If the material used for the side of the can costs $\$0.15/\text{cm}^2$ and the material for the top and bottom costs $\$0.20/\text{cm}^2$, what is the ratio of height to radius that results in the minimum cost.

18. A Norman window has a shape of a rectangle with a semi-circular top. The diameter of the semi-circle is the same as the width of the rectangle.

    (a) For a given perimeter, find the ratio of height to radius that will maximize the area of the window.

    (b) Because it takes additional time to fabricate a semi-circular top, it costs three times more per meter than the rectangle. For a given area, what ratio of height to radius would minimize the cost?

Answers:

1. 80 km/h, $2
2. 75×50 m
3. 4×4×4 m
4. 2.1 m^2
5. 4×4 cm
6. $r = 10\sqrt{2}$, $h = 20$
7. $w = \frac{8\sqrt{3}}{3}$, $h = \frac{32}{3}$
8. $(3, \frac{9}{\sqrt{2}})$, $d = \frac{9\sqrt{38}}{2}$
9. 384.5 m on land
10. 16.5 km from highway
11. 1.86 m
12. 45 km, 11:05:24
13. 1500, $180k
14. 1200
15. 687.5 m on land
16. $(\frac{1}{2}, \frac{5}{2})$
17. $\frac{h}{r} = \frac{8}{3}$
18. $\frac{h}{r} = 1$, $\frac{h}{r} = \pi + 1$